Metric conditions implying annular chaos
(Work in progress together with Fabio Tal)

Alejandro Passeggi

Abstract
One motivation behind this work is to set a bridge between different studies of chaos in Dynamical Systems.
1- On the one hand are the rigorous proofs of systems being chaotic, with the paradigmatic case of annular dynamics: in this case there are geometric models of chaotic examples, but it is usually hard to rigorously prove whether a parameter family of systems is chaotic or not, for example families coming from physics. Here we have two renowned cases: a Hamiltonian one given by the Poincaré section of the restricted three body problem, and a dissipative case given by the van der Pol oscillator (and similar systems). In both cases one works with parameters close to the integrable (non-chaotic) case, and shows that the perturbation exhibits homoclinic intersection.
In the latter, its simulations were a very important tool in order to then develop the theory of chaos. Usually this sort of rigorous analysis is hard to implement far from the integrable case. There exists some number of results given sufficient conditions to decide if a system is chaotic out of topological properties of the map, but are still far from being applicable to prescribed families of maps.
2- On the other hand there is a vast number of articles where the chaos is not rigorously obtained, but pictured by simulations of systems, whose motivation has its origin in sciences like physics, biology, chemistry, economy. However, these results are, as we said, mostly non-rigorous. Several of these simulations are taken over the dynamics of a map to the annulus, being the Poincaré return map of the original ODE.
The aim of the talk is to review the literature in pure math under the focus we just introduced, and present some partial results in the direction of setting a bridge which could give rigorous proofs of the existence of chaos out of simulations.