Some long standing problems in celestial mechanics and their resolution by computer assisted proof

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Abstract
Mathematical problems coming from celestial mechanics are notoriously challenging. The nonlinearities are strong, solutions may or may not exist for all time due to collisions, there are no attractors or repellors to organize the phase space, and the interesting questions are usually global in nature. Thanks to these features, progress in celestial mechanics has for several hundred years required the development and refinement of ever newer and more powerful tools. In the Twentieth Century major breakthroughs in variational methods, perturbative arguments, and global analysis (for example the equivariant degree theory) went hand and hand with breakthroughs in celestial mechanics.
The Twentieth Century witnessed both the explosion of digital computing and the inauguration of the space race. Again, it is fair to say that these two revolutions were strongly coupled, with advances in one driving advances in the other. By the 1990's, mathematicians began to investigate ways in which all of this new computing power could be used to prove theorems in celestial mechanics, with a special emphasis on parameter regimes far from any perturbative setting. This talk will begin with a general overview of these developments, followed by a rapid discussion of the history of computer assisted proof in this area. Finally I will focus on some more specific problems in celestial mechanics which have recently been solved by Maciej Capinski, Shane Kepley, Maxime Murray, and myself using computer assisted methods.