Bifurcation analysis of a model of tuberculosis epidemic allowing treatment of wider population suggesting a possible role in the seasonality of this disease

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Abstract
A novel epidemiological model describing the evolution of tuberculosis in a human population is proposed. This model is of the form SEIR, where S stands for Susceptible people, E for Exposed (infected but not yet infectious) people, I for Infectious people, and R for Recovered people. The main characteristic of this model inspired from the disease biology and some realistic hypothesis is that the treatment is administered not only to infectious but also to exposed people. Moreover, this model is characterized by an open structure, as it considers the transfer of infected or infectious people to other regions more conducive to their care and accepts treatment for exposed or infectious patients coming from other regions without care facilities. Stability and bifurcation of the solutions of this model are investigated. It is found that saddle-focus bifurcation occurs when the contact parameter β passes through some critical values. The model undergoes a Hopf bifurcation when the quality of treatment r is considered as a bifurcation parameter. It is shown also that the system exhibits saddle-node bifurcation, which is a transcritical bifurcation between equilibrium points. Numerical simulations are done to illustrate these theoretical results. Amazingly, the Hopf bifurcation suggests an unexpected and never suggested explanation of seasonality of such a disease, linked to the quality of treatment.

Keywords: epidemiological model, tuberculosis, bifurcation, dynamical systems, tuberculosis seasonality